

A different interpretation of "Measuring propagation speed of Coulomb fields" by R. de Sangro, G. Finocchiaro, P. Patteri, M. Piccolo, G. Pizzella

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Received: date / Accepted: date

Abstract We claim that the anti-relativistic statement in Ref. [1] that the Coulomb field of a moving charge propagates rigidly with it, cannot as a matter of fact be inferred from the measurements reported in that reference. Registered is not the passing of the Coulomb disk, but the acceleration-dependent part of the Liénard-Wiechert field.

Keywords moving charge · rigid propagation

According to the generally accepted Feynman's view [2] a certain time is needed for a charge to gain its disk-like shape prescribed by the Liénard-Wiechert formulas [3,4] ¹ after this charge starts its homogeneous motion (see [6]). In contrast with this view fully respecting the finiteness of the speed of propagation of interaction, the experimental results [1] obtained in Frascati National Laboratory on measuring the electric field of a charge injected into the working space in a certain point and then left to move freely with the speed v of the order of that of light, $v = c(1 - 0.5 \cdot 10^{-6})$, are interpreted by their authors as witnessing in favour of instantaneous propagation. This interpretation already called into being some speculations [7] about superluminal effects and causality violation. We are going to explain here that this interpretation can be avoided by taking into account the acceleration phase of the charge.

Let us imagine first that the charge is point-like and that it is emitted in the point $z' = 0$, $y' = 0$ and then moves along the axis z following the law

$$z' = vt', \quad y' = 0, \quad (1)$$

with z' and $y' = 0$ being the coordinates of its position at the time instance t' . The field is observed at the time-

¹It was explicitly demonstrated in [5] that these formulas and the one given in [2] are identical

instance t in the point z , y provided that it has been created by the charge when it was in the point z' and $y' = 0$ at the time t' if the "light cone" equation

$$(z - z')^2 + y^2 = c^2 (t - t')^2, \quad (2)$$

is obeyed, since the electromagnetic interaction propagates exactly with the speed of light c .

The important circumstance necessary to understand the experiment, rightfully emphasized by the authors, is that the disk accompanying the fast moving charge is so narrow that the field can be registered only at its maximum, otherwise it is many orders of magnitude smaller and beyond the experimental sensitivity. The maximum of the field passes the point z , y at the observation time

$$t = \frac{z}{v}, \quad (3)$$

as it follows from the form of the dominator, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

$$R^{*3} = \left[(z - vt)^2 + \frac{y^2}{\gamma^2}\right]^{\frac{3}{2}} \quad (4)$$

in the expression for the electric field of a moving charge, see Eq. (38,6) in [3] and also Eq. (6) below. Using (1) and (3) we obtain Eq. (6) in Ref. [1]

$$t' = t - \frac{y}{c}\gamma \quad (5)$$

as solution to equation (2) subject to the retardation condition $t' < t$.

In the experiment, the sensors were located in the points, whose coordinate z varied from 1 to 5 m, and y from 3 to 55 cm. With these values substituted into (5), (3) one gets the vast negative value for the time difference $t' - t$ (up to -1800 ns), which means that the

point $z' = z - y\gamma$ where the registered field has been created, is separated from the observation point by the tremendous negative distance up to $z' = -y\gamma = -550$ m ($z \ll y\gamma$). However, the distance of hundred meters for the beam to move before it is registered is not available in the experiment. In other words, it comes out that the field had been paradoxically created long before its source appeared. In short, we face a "disproof" of Special Relativity by *reductio ad absurdum*, because the above consideration is based on its postulates. The authors of Ref. [1] suggest to resolve the paradox by concluding that the charge appears with its disk field already formed, in other words, that the Coulomb field propagates rigidly together with the charge carrying it. This conclusion is, however, anti-relativistic in itself, since it admits forming the field in the whole space at once ².

To refute this disproof, it is necessary to consider a fuller problem of the charge being accelerated before it gets into the registration space (experiment hall). Under the acceleration, either the speeding up of the beam or its magnetic bending may be understood.

The full Liénard-Wiechert formula [3, 4] for accelerated charge is

$$\mathbf{E}(\mathbf{r}, t) = e \frac{1 - \frac{v^2}{c^2}}{(R - \frac{\mathbf{R}\mathbf{v}}{c})^3} \left(\mathbf{R} - \frac{\mathbf{v}}{c} R \right) + \quad (6)$$

$$+ e \frac{\left[\mathbf{R} \times \left[\left(\mathbf{R} - \frac{\mathbf{v}}{c} R \right) \times \frac{\dot{\mathbf{v}}}{c^2} \right] \right]}{(R - \frac{\mathbf{R}\mathbf{v}}{c})^3}, \quad (7)$$

where $\dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt}$ is the acceleration of the charge at the moment when it created the electromagnetic field to be registered at the time t in the point (z, y) . The vector $\mathbf{R} = (z - z', y)$ is drawn from the point $(z', 0)$, where the field is created to the observation point (z, y) . Its modulus $|\mathbf{R}| = R = c(t - t')$ is the distance between these points in agreement with (2). The identity of R^{*3} (4) with the denominators in (6) is seen taking into account that $\mathbf{R}\mathbf{v} = v(z - z')$. Then

$$R - \frac{\mathbf{R}\mathbf{v}}{c} = c(t - t') - \frac{v}{c}(z - z'),$$

which coincides with R^* after the substitution of the light-cone condition (2) and the charge trajectory (1) into (4). The vector $\mathbf{R} - \frac{\mathbf{v}}{c} R = \mathbf{R} - \mathbf{v}(t - t') =$

$= (z - z' - v(t - t'), y) = (z - vt, y)$ in the numerators of (6) is the vector drawn from the point $(vt, 0)$ where the charge is ³ at the moment of observation to the observation point (z, y) .

The first term in (6) is suppressed by the factor $\gamma^{-2} = 1 - \frac{v^2}{c^2} = 10^{-6}$. (This fact has forced the authors of Ref. [1] to admit that the field of the moving charge might be registered only in the maximum point 3). On the contrary, the second term does not contain this small factor. To estimate its value let us assume that the beam has been turned by the bending magnets short before the injection as it is characteristic of the Beam Test Facility in the Frascati National Laboratory, and that the curvature radius is of the same centimeter scale as R . Then the centrifugal acceleration is $|\dot{\mathbf{v}}| = \frac{v^2}{R}$, and the extra factor in the second term in (6) as compared with the first term is of the order of unity: $R \frac{\dot{\mathbf{v}}}{c^2} = R \frac{1}{c^2} \frac{v^2}{R} \approx 1$, not suppressed by the factor 10^{-6} present in the first term. This may signify that it is the field given by the second, radiative term in (6) that is registered in the experiment, and, moreover, not in the point of its maximum, but when it first appears, being created at the time of acceleration. If, quite roughly, we imagine, following Ref. [6], that the charge was immediately accelerated in the point $z' = y' = 0$ at the time instance $t' = 0$, the spherical wave is created propagating with the speed of light c . It reaches the sensors located at the points $z = z_1, z_2, y = y_1, y_2$ at times $t = t_1, t_2$, respectively, with $c^2 t_{1,2}^2 = z_{1,2}^2 + y_{1,2}^2$. The observed longitudinal "speed" V_{long} of the registered signal between these points defined as (we mean $z_1 < z_2, t_1 < t_2$), the distance along the z -axis ($z_2 - z_1$) divided by the difference of the times ($t_2 - t_1$), at which the wave reaches the two sensors, is

$$V_{\text{long}} = \frac{c(z_2 - z_1)}{(y_2^2 + z_2^2)^{\frac{1}{2}} - (y_1^2 + z_1^2)^{\frac{1}{2}}}. \quad (8)$$

This is not determined by the speed of the beam, which is ever smaller than c . Contrary to the latter, V_{long} may be larger than c (without contradicting principles of Special Relativity, since no information can be transmitted with this "speed"). Besides, V_{long} is not a universal, but depends upon the choice of the sensor positions. If $y_1 = y_2 = y$ the following chain of inequalities, obtained each by squaring the previous one

$$\begin{aligned} z_2 - z_1 &> (y^2 + z_2^2)^{\frac{1}{2}} - (y^2 + z_1^2)^{\frac{1}{2}}, \quad (9) \\ (y^2 + z_2^2)^{\frac{1}{2}} (y^2 + z_1^2)^{\frac{1}{2}} &> y^2 + z_1 z_2, \\ z_2^2 + z_1^2 &> 2z_1 z_2, \end{aligned}$$

³to be more precise, where it would be if it continues to move with the same constant speed after it has created the observed field.

²The natural view [6] is that only the microscopic core of the charge, where its field mass is mostly gained (for instance, due to its nonlinear self-interaction [8]) is an integral part of it and thus may be thought of as ever accompanying it. As long as the electron is concerned, this is its classical radius of the order of 3 fm. Certainly, the microcausality is questioned within this assumption. However, the present measurements do not deal with such small distances.

proves that $V_{\text{long}} > c$ in such case. If next we go to larger values of y_1 , $y_1 > y_2$, the inequality $z_2 - z_1 > (y_2^2 + z_2^2)^{\frac{1}{2}} - (y_1^2 + z_1^2)^{\frac{1}{2}}$ becomes even stronger than what it was when $y_1 = y_2$, (9). The "speed" V_{long} becomes larger and larger, till y_1 reaches the root of the equation $(y_2^2 + z_2^2)^{\frac{1}{2}} - (y_1^2 + z_1^2)^{\frac{1}{2}} = 0$, where the denominator in (8) turns to zero. In this case the wave comes to the both sensors simultaneously, hence it is no wonder that $V_{\text{long}} = \infty$. So, we conclude that $V_{\text{long}} > c$ at least as long as $y_1 \geq y_2$.

Let us now turn to the results of measuring the longitudinal "speed" as these are presented in Table 1 of Ref. [1]. The first remark concerning this Table is that even taking into account the indeterminacies stemming from the finite sizes of the sensors one cannot state – contrary to what the authors do – that the speeds between different couples of sensors are one speed, as these should have been if the speed were that of the beam. The second remark is that the latter must be smaller than (almost equal to) c , whereas the results listed in Table 1 are at least in certain cases definitely above the speed of light. Unfortunately, the values of the y -positions of the sensors, between which the speeds were measured, are not indicated by the authors, and this fact makes it impossible to imply the model formulas (8). Nevertheless, if we calculate the speed following the data given in the middle column of the first line of Table 1 we get

$$V_{\text{long}} = \frac{223.0 \pm 1.5 \text{ cm}}{7.28 \pm 0.02 \text{ ns}}.$$

This ranges within $(3.092 \div 3.033) \cdot 10^{10} \text{ cm/s}$, which is wholly and essentially above the speed of light. The value $V_{\text{long}} = 3.063 \cdot 10^{10} \text{ cm/s}$ supplied by Eq. (8) with the values $z_1 = 329.5 \text{ cm}$, $z_2 = 552.5 \text{ cm}$, $y_1 = 55 \text{ cm}$, $y_2 = 5 \text{ cm}$ fits this range.

To be convinced that they are really registering the coming of the Coulomb field to their sensors, the authors of [1] performed the background measurements, when a filter of lead was placed in the way of the charge before it might reach the projection points z of the sensors. The results presented in their Fig. 15 indicate that the countings are much smaller in that case. However, this fact cannot be taken as a proof that really the passing of the disk is registered when there is no filter, because after the Coulomb field is formed at time t' it continues to exist afterwards irrespective of the consequent fate of the charge, i.e. even after it is absorbed by the filter. The results of the background measurements can be only understood as an indication that not solely the charge is absorbed by the lead filter, but also its

Coulomb field, as well as the radiation field. An indirect confirmation to this assumption may be in that the counting in Fig. 15 seem somewhat growing with the growth of the transversal position of the sensor y , when the admitted screening may be expected to become less efficient. This assumption is favoured by the fact that the counting for the sensor most remote from the beam axis, $y = 55 \text{ cm}$, is practically the same irrespective of whether the filter is present.

Our conclusion is that most probably the signal registered in the Frascati experiment [1] originates from the radiation due to the acceleration of the beam, and does not belong to the Coulomb disk of the charge. As for the latter, it should be sought for hundreds of meters ahead, already outside of the laboratory, after the beam itself is absorbed by its concrete wall, unless, certainly, it is screened by other possible objects.

Acknowledgements Supported by RFBR under Project 14-02-01171, and by the TSU Competitiveness Improvement Program, by a grant from "The Tomsk State University D.I. Mendeleev Foundation Program".

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